

Testing lepton number violation with the reaction  $e^-e^- \rightarrow \mu \nu q \bar{q}$ J. Maalampi<sup>a</sup> and N. Romanenko<sup>a,b</sup><sup>a</sup> *Theoretical Physics Division, Department of Physics, University of Helsinki, Finland*<sup>b</sup> *Petersburg Nuclear Physics Institute, Gatchina, Russia***Abstract**

We investigate the reaction  $e^-e^- \rightarrow \mu \nu q \bar{q}$  as a possible place to test the lepton number violating interactions of singly charged scalars  $\Delta_L^\pm$  belonging to a  $SU(2)_L$  triplet. The existence of such scalars is predicted by some majoron models, as well as by the left-right symmetric extension of the Standard Model. We show that this reaction may be observed in  $e^-e^-$  collision well below the threshold of  $\Delta_L^\pm$ . For the colliding energy  $\sqrt{s} = 500$  GeV the mass of the singly charged Higgs triplet may be excluded up to 2 TeV and even more, depending on values of the appropriate Yukawa couplings.

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Recent experimental results of the SuperKamiokande collaboration on atmospheric neutrinos [1] strongly indicate that neutrinos oscillate and consequently that at least some of neutrino species have nonzero mass. This gives rise to a series of interesting questions concerning the origin and nature (Dirac/Majorana) of those masses. Specifically, it would be interesting and important to know the structure of Higgs sector responsible for the indicated mass and mixing pattern. For that, it is obviously necessary to go beyond the Standard Model (SM) as in the SM neutrinos are massless. If one does not introduce right-handed neutrinos, one possibility is to extend the scalar sector of the SM with an  $SU(2)_L$  triplet ( $\Delta_L$ ) Higgs with hypercharge  $Y = 2$ . This could yield Majorana mass to left-handed neutrinos through spontaneous symmetry breaking, assuming that  $\langle \Delta_L \rangle \neq 0$ . This is what happens in some majoron models [2], and the triplet Higgses quite naturally appear also in the left-right symmetric model (LR-model) of electroweak interactions [3, 4].

The Higgs triplet  $\Delta_L$  with hypercharge  $Y = 2$  consists of a doubly charged scalar, a singly charged scalar and two neutral scalars. The gauge invariance allows the triplet  $\Delta_L = (\Delta_L^0, \Delta_L^-, \Delta_L^{--})$  to interact with leptons via a Yukawa coupling that violates the lepton number  $L$  by two units. Due to this property the search of these particles is particularly handy and advantageous in electron-electron collisions where one has  $L = 2$  in the initial state.

The phenomenology of the doubly charged Higgs particle has been studied, e.g., in [5]. It was shown that  $\Delta_L^{--}$  with a mass up to 800 GeV may be excluded at the coming LHC experiments. In the present paper we shall concentrate on the search of the singly charged member of the Higgs triplet,  $\Delta_L^-$  in  $e^-e^-$  collisions at a linear collider [6]. Obviously, due to their different charges, the phenomenological signatures of the singly charged triplet Higgs differ considerably from those of the doubly charged Higgs. The present experimental data allows to restrict the mass of the singly charged bosons to be above roughly 100 GeV [7]. It may thus happen that the singly charged triplet Higgs turns out to be considerably lighter than its doubly charged counterpart. Hence it is worthwhile to examine its experimental signatures and prospects for obtaining information on its properties independently of the properties of the doubly charged scalar.

There is an important restriction that affects the production and decay rates of the triplet Higgs  $\Delta_L$ . The vacuum expectation value of its neutral member,  $\langle \Delta_L^0 \rangle =$

$v_L/\sqrt{2}$ , is limited to quite small values in order to avoid a violation of the experimentally well established relation  $\rho \equiv M_W^2/(M_Z^2 \cdot \cos^2 \theta_W) = 1$ . This restriction remains the same for the LR-model if the  $W_L - W_R$  and  $Z_L - Z_R$  mixings are neglected [8]. The present experimental data indicate that  $v_L \leq 15$  GeV. Let us note, that there is an option to extend the Higgs sector further so that the equality  $\rho = 1$  holds at the tree level due to the so-called "custodial"  $SU(2)$  symmetry [9]. In this scenario, which will not be considered in the following, there would be more than one charged triplet scalars.

In this paper we will show that the process  $e^- e^- \rightarrow \mu \nu q \bar{q}$  provides a good environment to study the interactions of the singly charged scalar  $\Delta_L^-$ , quite independently of the properties of the doubly charged Higgs  $\Delta_L^{--}$ . For the anticipated luminosity of electron-electron collider this process may be observed well below the production threshold of the singly charged Higgs.

We consider the standard  $SU(2)_L \times U(1)_Y$  model with an additional Higgs triplet field of hypercharge  $Y = 2$ :

$$\Delta_L = \begin{pmatrix} \Delta_L^+/\sqrt{2} & \Delta_L^{++} \\ \Delta_L^0 & -\Delta_L^+/\sqrt{2} \end{pmatrix} = (3, 1, 2) \quad (1)$$

with the vacuum expectation value:

$$\langle \Delta_L \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix}. \quad (2)$$

This kind of model was first suggested in [2] in order to generate Majorana masses for neutrinos, and it also quite naturally arises as an effective low-energy manifestation of the left-right symmetric model [10]. The couplings of  $\Delta_L$  with the gauge fields are provided by the usual kinetic term in the Lagrangian,

$$L_{\text{kin}} = \frac{1}{2} \text{Tr} (D_\mu \Delta_L)^+ (D_\mu \Delta_L), \quad (3)$$

where

$$D_\mu \Delta_L = \partial_\mu \Delta_L + ig' B_\mu \cdot \Delta_L + \frac{ig}{2} W_\mu^a [\tau^a, \Delta_L], \quad (4)$$

and its interactions with fermions are given by a Yukawa coupling of the form

$$L_{\text{Yuk}} = -h_{L,\alpha\beta} \Psi_{\alpha L}^T C \sigma_2 \Delta_L \Psi_{\beta L} + \text{h.c.}, \quad (5)$$

where  $\Psi$  denotes the lepton doublet  $(\nu_l, l^-)$  and  $\alpha$  and  $\beta$  are flavour indices. The Yukawa interaction gives rise to Majorana mass terms  $m_{\nu_{\alpha\beta}} = h_{L,\alpha\beta} \cdot v_L$  for the left-handed neutrinos.

The interactions of the  $\Delta_L$  field described above in (3), (5) are the same for both the SM with additional Higgs triplet and the LR-model. Of course, the full Lagrangian should also include self-interactions of the scalar fields, which are governed by the respective scalar potentials. We will overlook these in the following as a first approximation by assuming no mixing between the doublet and triplet Higgses and the absence of any further Higgs fields.

The phenomenologically most important feature of the models that include triplet Higgses is the lepton number non-conservation arising from the Yukawa coupling (5). It makes the interactions of the triplet Higgses quite complementary to the Yukawa interactions of the SM Higgs doublet, and it provides good opportunities for unambiguous tests of  $\Delta_L^-$  production and decay in  $e^-e^-$  collision experiments. The production of like-sign charged Higgs bosons via charged vector boson fusion in several electroweak models was considered in [11]. It was shown that cross section strongly depends on the choice of the Higgs representation and on the parameters of the model. The models described above were, however, not considered in that study. It is impossible to distinguish the lepton number conserving properties of charged Higgs through this kind of process.

In [11] the production of the singly charged Higgses in  $e^-e^-$  collisions was assumed to happen in pairs through a  $W^-W^-$  fusion. This process conserves the lepton number. We are interested, in contrast, in processes that probe the lepton number violating Yukawa couplings (5). The pair production, which proceeds through t-channel exchange of Majorana neutrinos and s-channel exchange of  $\Delta_L^{--}$ , is in this case not a suitable process to study, however. This is because the neutrino exchange is proportional to Majorana mass of the neutrino and hence is suppressed and the  $\Delta_L^{--}\Delta_L^-\Delta_L^-$  vertex depends on the self-couplings of scalar potential whose values are unknown. We consider instead a production of a single  $\Delta_L^-$  in the process  $e^-e^- \rightarrow \Delta_L^- W_\mu^-$  which proceeds through t-channel exchange of Majorana neutrino and s-channel exchange of  $\Delta_L^{--}$ . In this case the neutrino exchange is not suppressed as the t-channel neutrino has the same chirality in the both vertices. Moreover, in the s-channel process the strength of the  $\Delta_L^{--}\Delta_L^-W_\mu^-$  vertex does not depend on any unknown parameter of the scalar potential but is determined by the gauge coupling. The experimentally clearest final state to study is the one where  $\Delta_L^-$  decays to a muon and a muonic neutrino and  $W_\mu^-$  decays into two quark jets without missing energy. We hence choose the process

$e^-e^- \rightarrow \mu\nu q\bar{q}$  for further investigation, specifically with the light quarks  $d$  and  $\bar{u}$  in final state.

In addition to the two amplitudes mentioned above, there exist still another amplitude contributing in the process  $e^-e^- \rightarrow \mu\nu q\bar{q}$ , namely the one where  $\Delta_L^{--}$  is exchanged in s-channel producing two muons, one of which decaying further into  $\nu d\bar{u}$ . This amplitude does not directly depend on the properties of  $\Delta_L^-$  and is for that reason undesirable, but its influence is, of course, unavoidable.

In our calculations, whose results will be presented in the following, we have used CompHEP package created in Moscow University [12]. As an input we have used the vertex functions  $\sqrt{2} \cdot h_{L,ee} \cdot (1 - \gamma_5)/2$  and  $-g \cdot (p_\mu^{++} - p_\mu^-)$  for the  $e\nu_e\Delta_L^+$  and  $W_L^-\Delta_L^{++}\Delta_L^-$  couplings, respectively, as well as the ordinary electroweak vertices for the couplings of  $W^-$  with leptons and quarks. In our calculations we have imposed the following cuts for the final state phase space:

- Each final state particle has energy greater than 10 GeV (including neutrino).
- The transverse energy of each particle (including missing transverse energy) should be greater than 5 GeV.
- The opening angle between two quark jets should be more than  $20^\circ$ .
- Each final state particle should have the outgoing direction more than  $10^\circ$  away from the beam axis.

In Fig. 1 we present the dependence of the cross section of the process  $e^-e^- \rightarrow \mu\nu d\bar{u}$  on the collision energy for the different values of masses of singly ( $M_{\Delta_L^-} = 100, 400, 700, 1000$  GeV) and doubly charged ( $M_{\Delta_L^{--}} = 100, 400, 700, 1000$  GeV) triplet Higgses. The cross sections are dominated by the resonance at  $\sqrt{s} = M_{\Delta_L^{--}}$ . To estimate the width of the peak we have chosen  $\Gamma_{\Delta_L^{--}} = 10^{-3} \cdot M$  for the two lepton decays and the  $\Delta_L^{--} \rightarrow \Delta_L^- W_L^-$  mode was also taken into account [5]. One may conclude that at 0.01 fb level the process  $e^-e^- \rightarrow \mu\nu d\bar{u}$  may be observed away from the  $\Delta_L^{--}$  resonance and even below the  $\Delta_L^-$  threshold.

In Fig. 2 presents the sensitivity of the reaction  $e^-e^- \rightarrow \mu\nu d\bar{u}$  on the masses of the triplet Higgs particles  $\Delta_L^-$  and  $\Delta_L^{--}$  for the collision energy  $\sqrt{s} = 500$  GeV. We have estimated the values of the running coupling constants at 500 GeV by applying the approximate RG equations of the SM [13]. The influence of the triplet Higgses on the running, which can be expected to be quite small, is not taken into account.

Fig. 2a displays the cross section of the  $e^-e^- \rightarrow \mu\nu d\bar{u}$  process for the different values of the  $\Delta_L^-$  and  $\Delta_L^{--}$  masses, with assuming for the Yukawa couplings their maximal allowed values that are in accordance with the present phenomenological constraints [5, 14]:

$$\begin{aligned} h_{ee}^2 &< 10^{-5} \cdot M_{\Delta_L^{--}} \text{ GeV}, \\ h_{\mu\mu}^2 &< 10^{-5} \cdot M_{\Delta_L^{--}} \text{ GeV}. \end{aligned} \quad (6)$$

If the mass of doubly charged Higgs is considered to be greater than 100 GeV, then  $h_{ee} \cdot h_{\mu\mu} < 0.18$  or  $\sqrt{h_{ee} \cdot h_{\mu\mu}} < 0.4$ .

As can be seen from Fig. 2a, the process  $e^-e^- \rightarrow \mu\nu d\bar{u}$  will be well observable at colliding energy 500 GeV for any values of  $\Delta_L^-$  and  $\Delta_L^{--}$  masses below 2 TeV if the Yukawa couplings of the triplet fields have their maximal allowed values. Furthermore, the anticipated luminosity of linear collider (0.03 fb at  $\sqrt{s} = 500$  GeV [6]) allows to state that the value of the product  $h_{ee} \cdot h_{\mu\mu}$ , which enters as a common factor in all considered Feynman amplitudes, may be restricted at least one order of magnitude better than the present bound. Nevertheless, it is not so easy to separate the influence of the singly charged Higgs from that of the doubly charged Higgs particle. First of all let us notice, that in the limit of  $M_{\Delta_L^-}$  going to infinity, the cross section remains finite due to contribution of the s-channel process  $e^+e^- \rightarrow \Delta_L^{--} \rightarrow \mu^+\mu^-$  followed by subsequent decay of one of the muons to a  $\nu d\bar{u}$ . This process does not involve the singly charged Higgs. If  $\Delta_L^{--}$  in turn is decoupled, the amplitude with the t-channel exchange of Majorana neutrinos would keep the cross section finite. In order to search for the effects of the singly charged triplet Higgs we have studied the difference between the cross sections of the  $e^-e^- \rightarrow \mu\nu d\bar{u}$  process at a fixed and an infinite value (in practice  $M_{\Delta_L^-} = 2$  TeV) of the  $\Delta_L^-$  mass. In Fig. 2b we show the dependence of the cross section on the  $\Delta_L^{--}$  mass in the case that  $\Delta_L^-$  is effectively decoupled. Supposing that the mass of  $\Delta_L^{--}$  is known one can conservatively estimate, by setting for the Yukawa coupling  $h_{\mu\mu}$  the largest phenomenologically allowed value, the contribution of the  $\Delta_L^{--}$  mediated processes on the total cross section. When this is subtracted from the total cross section, what is left is the contribution of the t-channel neutrino exchange process alone. This has a threshold behaviour and its strength gives direct information on the product  $h_{ee} \cdot h_{\mu\mu}$  of Yukawa couplings.

In Fig. 2c we display the 0.03 fb (30 events per year) discovery contours on the  $(M_{\Delta_L^-}, M_{\Delta_L^{--}})$ -plane, corresponding to the cross section of the isolated t-channel

process, for the different values (0.1, 0.4 and 1.0) of "average" Yukawa couplings ( $h_{\text{Yuk}} = \sqrt{h_{ee} \cdot h_{\mu\mu}}$ ). In the plot the collision energy is taken as  $\sqrt{s} = 500$  GeV. It is seen from the figure that the process  $e^-e^- \rightarrow \mu\nu d\bar{u}$  might probe the the mass  $M_{\Delta_L^-}$  to much larger values than what is the production threshold, providing that the average Yukawa coupling is larger than 0.1 and the collision does not happen in the vicinity of the  $\Delta_L^{--}$  pole. If these conditions are not met  $\Delta_L^-$  would have detectable effects only when it is produced as a real particle.

The constraints ensuing for the average Yukawa coupling are presented in Fig. 3 for different values of the mass of the doubly charged Higgs ( $M_{\Delta_L^{--}} = 400, 1000, 1500$  and 2000 GeV) at  $\sqrt{s} = 500$  GeV. For a light  $\Delta_L^{--}$  the constraint is tightest, about  $h_{\text{Yuk}} \lesssim 0.1$  and it does not depend on the mass of  $\Delta_L^-$  outside the threshold region. The larger  $M_{\Delta_L^{--}}$  the weaker and also the more dependent on the mass of  $\Delta_L^-$  is the bound.

The main SM background to the reaction  $e^-e^- \rightarrow \mu\nu d\bar{u}$  is due to the process  $e^-e^- \rightarrow W^-W^-\nu\nu$  studied in [15]. Its cross section was estimated to be below 20 fb for colliding energy  $\sqrt{s} \lesssim 1$  TeV. Taking into account the branching rations of  $W$  to the appropriate decay modes we can estimate the background to be about 1 fb. This is typically below the process  $e^-e^- \rightarrow \mu\nu d\bar{u}$  rate, but may be much larger than the singly charged contribution to the process. However, reconstructing the invariant squared mass of the muon and neutrino pair it would be possible to separate background in the cases when the mass difference between  $\Delta_L^-$  and  $W^-$  is greater than invariant mass resolution (for  $M_{\Delta_L^-} > 100$  GeV this should be possible). But even in the cases when  $M_{\Delta_L^-} \simeq M_W$  it is possible to compare the cross sections of  $e^-e^- \rightarrow \mu\nu d\bar{u}$  and  $e^-e^- \rightarrow d\bar{u} s\bar{c}$  which should be equal in the SM. Any substantial difference between these cross sections would be a signal of the new physics. In other words, in order to get rid of the SM background one should consider the ratio of the cross sections of  $e^-e^- \rightarrow d\bar{u} s\bar{c}$  and  $e^-e^- \rightarrow \mu\nu d\bar{u}$ .

In summary, we have shown that the process  $e^-e^- \rightarrow \mu\nu d\bar{u}$  provides a good test for lepton flavor non-conservation of the singly charged scalars. At the collision energy 500 GeV the process may be seen well below  $\Delta_L^-$  and/or  $\Delta_L^{--}$  thresholds for a wide range of the lepton number violating Yukawa couplings. The influence of  $\Delta_L^-$  contribution (below its threshold) may be extracted from the process, if colliding energy is away from the  $\Delta_L^{--}$  resonance. The present bounds on the Yukawa couplings may

be significantly improved.

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## References

- [1] Y. Fukuda et. al., hep-preprint hep-ex/9805006 (1998).
- [2] G. Gelmini and M. Roncadelli, Phys. Lett. B 99 (1981) 411; H. Georgi, S. Glashow and S. Nussinov, Nucl. Phys. B193 (1981) 297.
- [3] J. C. Pati and A. Salam, Phys. Rev D10 (1974) 275; R. N. Mohapatra and J. C. Pati, Phys. Rev. D11 (1975) 2558; G. Senjanovic and R. N. Mohapatra, Phys. Rev. D12 (1975); R. N. Mohapatra and R. E. Marshak, Phys. Lett. B91 (1980); R. N. Mohapatra and D. Sidhu, Phys. Rev. Lett. 38 (1977) 667.
- [4] M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, eds. P. van Nieuwenhuizen and D. Z. Freedman (North Holland 1979); T. Yanagida, in Proceedings of *Workshop on Unified Theory and Baryon Number in the Universe*, eds. O. Sawada and A. Sugamoto (KEK 1979); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912.
- [5] M.L. Swartz, Phys. Rev. D 40 (1989) 1521; M. Lusignoli and S. Petrarca, Phys. Lett. B 226 (1989) 397; R. Mohapatra, Phys. Rev. D46 (1992) 2990; J. Gunion, Int. J. Mod. Phys A11 (1996) 1551; D. London, G. Belanger and J. Ng, Phys. Lett. B188 (1987) 155; K. Huitu, J. Maalampi, A. Pietilä and M. Raidal, Nucl.Phys. B487 (1997) 27.
- [6] E. Accomando et al., Phys. Rept. 299 (1998) 1.
- [7] Particle Data Group (R.M. Barnett et al.), Phys. Rev. D54 (1996) 1.
- [8] J.F. Gunion, J. Griffols, A. Mendez, B. Kayser and F. Olness, Phys. Rev. D40 (1989) 1546.
- [9] H. Georgi and M. Machacek, Nucl. Phys. B262 (1985) 463; S. Chanowitz and M. Golden, Phys. Lett. B165 (1985) 105.
- [10] N. G. Deshpande, J. F. Gunion, B. Kayser and F. Olness, Phys. Rev. D44 (1991) 837; P. Langacker and S. Uma Sankar, Phys. Rev. D40 (1989) 1569;
- [11] T.Rizzo, Int. J. Mod. Phys. A11 (1996) 1563.
- [12] P. A. Baikov et. al., in: *Proc. of X Workshop on High Energy Physics and Quantum Field Theory (QFTHEP-96)*, ed. by B. B. Levchenko and V. I. Savrin, Moscow, 1996, p. 101., hep-ph/9701412; E. E. Boos, M. N. Dubinin, V. A. Ilyin, A. E. Pukhov and V. I. Savrin, preprint INP MSU 94-36/358 and SNUCTP 94-116

(1994), hep-ph/9503280; A. Pukhov, E. Boos, M. Dubinin, V. Edneral, V. Ilyin D. Kovalenko, A. Kryukov, V. Savrin, S. Shichanin and A. Semenov, "CompHEP - a package for evaluation of Feynman diagrams and integration over multi-particle phase space", preprint INP-MSU 98-41/542, hep-ph/9908288.

- [13] A.A. Andrianov , P. Osland, A.A. Pankov, N.V. Romanenko and J. Sirkka, Phys. Rev. D58, (1998) 075001; J. Maalampi and N. Romanenko, Phys. Rev. D60 (1999) ( hep-ph/9810528).
- [14] G. Barenboim, K. Huitu, J. Maalampi and M. Raidal, Phys. Lett. B394 (1997) 132.
- [15] F. Cuypers, K. Kolodziej and R. Rueckl, Nucl. Phys. B430 (1994) 231.

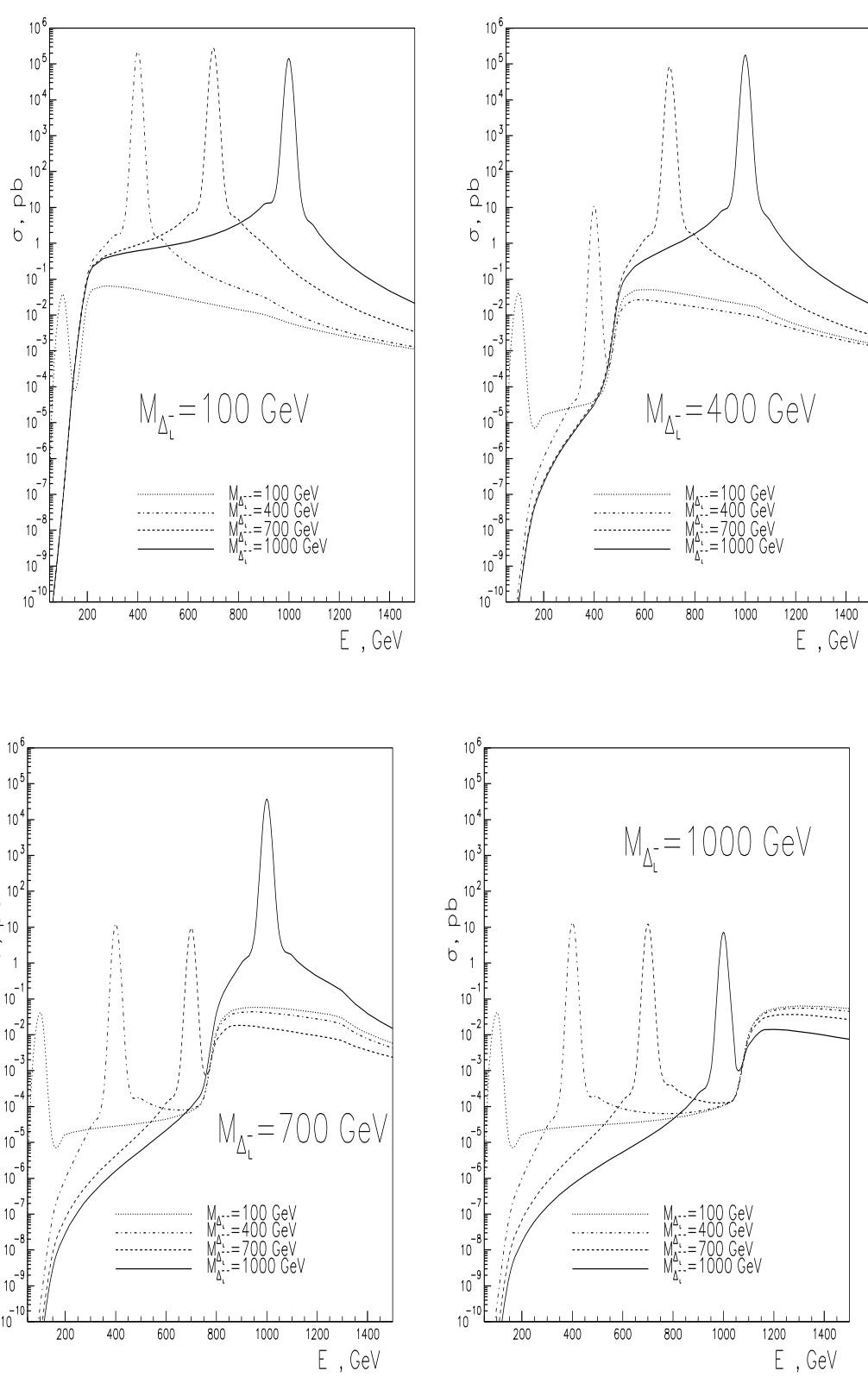
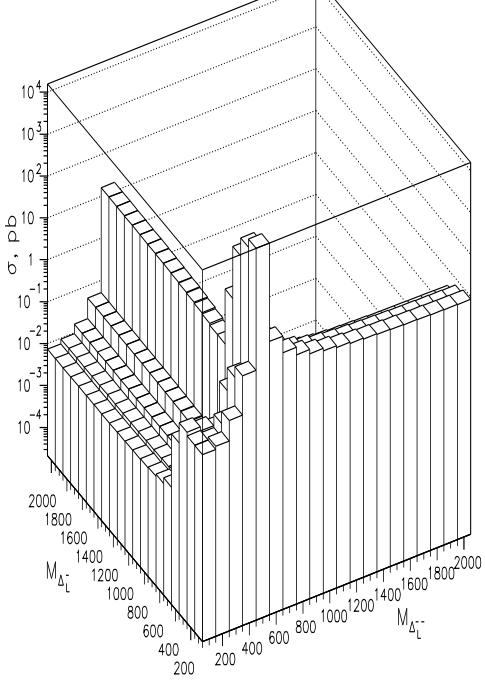
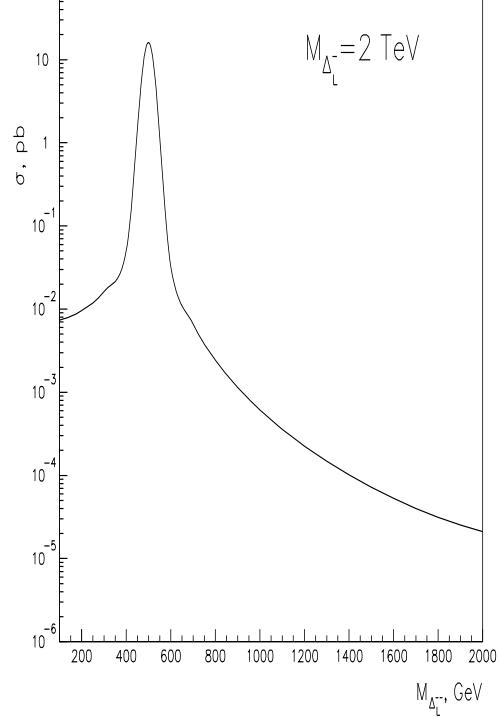


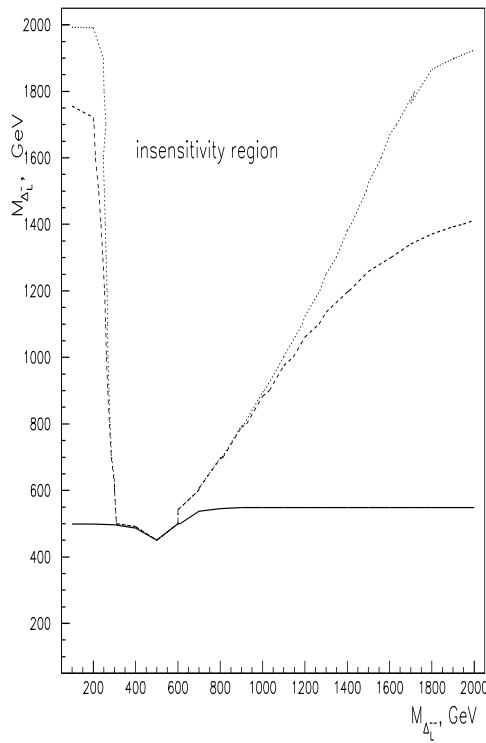
Figure 1: Energy dependence of the cross section of the  $e^-e^- \rightarrow \mu\nu_\mu d\bar{u}$  for different values of the masses of singly charged ( $\Delta_L^-$ ) and doubly charged ( $\Delta_L^{--}$ ) triplet Higgses.



(a)



(b)



(c)

Figure 2: The cross section for the  $e^-e^- \rightarrow \mu \nu_\mu d\bar{u}$  process (a) for the different values of  $M_{\Delta_L^-}$  and  $M_{\Delta_L^{--}}$ , (b) as a function of  $M_{\Delta_L^{--}}$  in the limit  $M_{\Delta_L^-} \gg M_{\Delta_L^{--}}$ . (c) The contour plots for the difference between cross sections of the process with finite and infinite  $M_{\Delta_L^-}$  for 0.03 fb (30 events per year), for different values of Yukawa couplings ( $h = 0.1$ , dashed line for  $h = 0.4$ , and dotted line for  $h = 1$ ). Collision energy is taken to be  $\sqrt{s} = 500$  GeV. In Fig. (c) the region above curves is outside the reach of experiment.

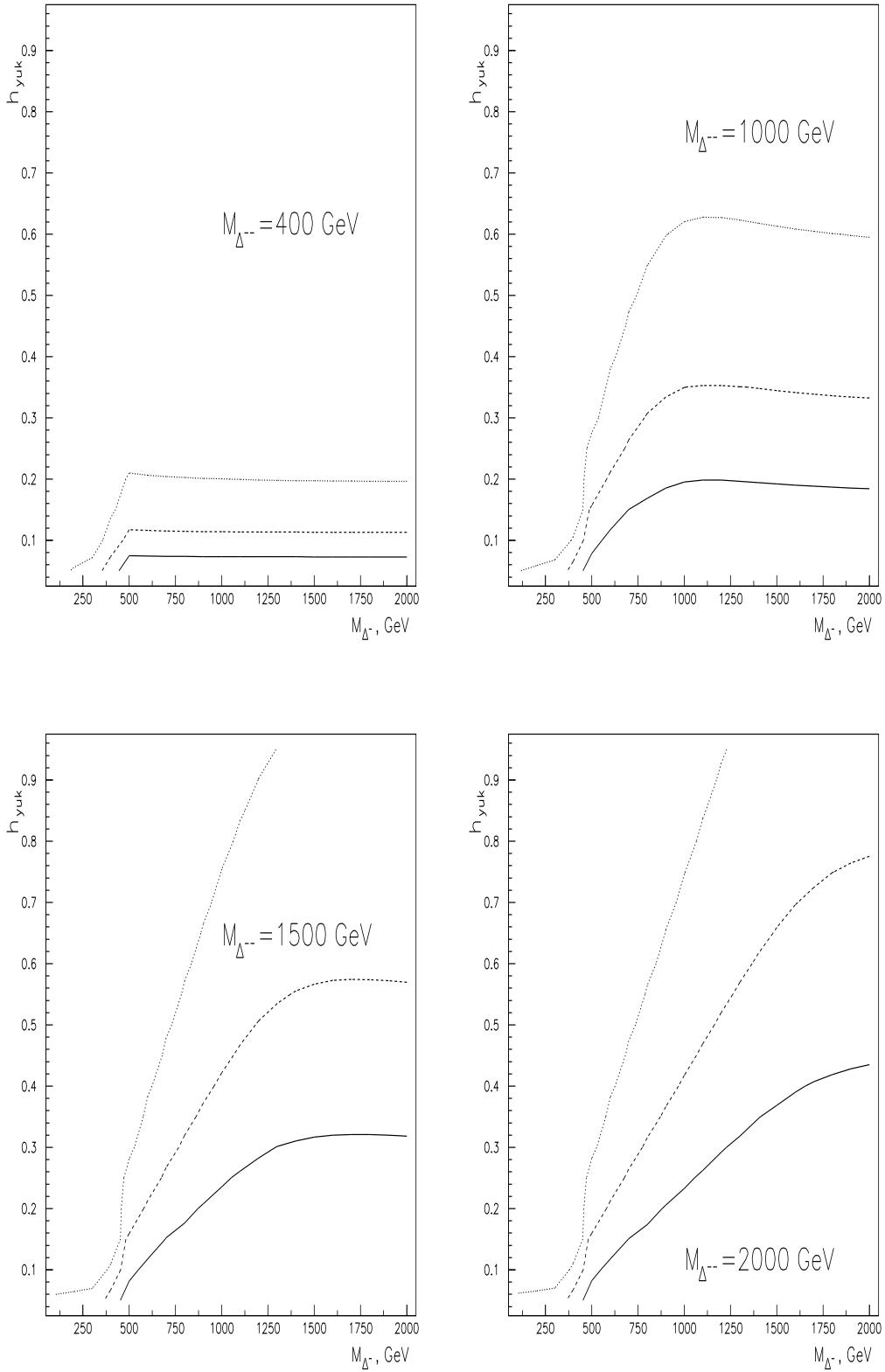


Figure 3: Sensitivity contours of the reaction  $e^-e^- \rightarrow \mu \nu_\mu d \bar{u}$  on the  $(h_{Yuk}, M_{\Delta^-})$  – plane corresponding to  $\sigma = 0.03$  fb (30 events per year, solid line),  $\sigma = 0.3$  fb (300 events per year, dashed line),  $\sigma = 3$  fb (30 events per year, dotted line) for different values of  $M_{\Delta_L^{--}}$ . Collision energy is taken to be  $\sqrt{s} = 500$  GeV.